Disjoint-set Operations

Kuan-Yu Chen (陳冠宇)

2019/05/07 @ TR-310-1, NTUST

Disjoint Sets.

- Some applications involve grouping n distinct elements into a collection of disjoint sets
  - Example:
    - $\{1, 2, 3\}$
    - $\{4, 5\}$
    - $\{6\}$
    - $\{1, 3, 5\}$
    - $\{1, 2, 3\}, \{4, 5\}, \{6\}$ are disjoint sets
    - $\{1, 2, 3\}, \{4, 5\}, \{6\}, \{1, 3, 5\}$ are not disjoint sets

Disjoint Sets..

- A ***disjoint-set data structure*** maintains a collection $\boldsymbol{S} = \{S_1, S_2, \dots, S_k\}$ of disjoint dynamic sets
  - We identify each set by a ***representative***
    - Representative should be time-invariant
    - How to choose the representative

- Letting x denote an object, an element in a set, we wish to support the following operations
  - MAKE-SET(x) creates a new set whose only member (and thus representative) is x
  - UNION(x, y) unites the dynamic sets that contain x and y into a new set
  - FIND(x) returns a pointer to the representative of the (unique) set containing x

Example.

- One of the many applications of disjoint-set data structures arises in determining the connected components of an undirected graph
  - Given a graph $G = (V, E)$, where $V = \{a, b, c, d, e, f, g, h, i\}$ and $E = \{(b, d), (e, g), (a, c), (h, i), (a, b), (e, f), (b, c)\}$
    - The procedure CONNECTED-COMPONENTS initially places each vertex v in its own set
    - Then, for each edge (u, v), it unites the sets containing u and v

```
CONNECTED-COMPONENTS(G)
for each vertex v ∈ G.V
    MAKE-SET(v)
for each edge (u, v) ∈ G.E
    if FIND-SET(u) ≠ FIND-SET(v)
        UNION(u, v)
```

Example..

– Given a graph $G = (V, E)$, where $V = \{a, b, c, d, e, f, g, h, i\}$ and $E = \{(b, d), (e, g), (a, c), (h, i), (a, b), (e, f), (b, c)\}$

```
CONNECTED-COMPONENTS(G)
for each vertex v ∈ G.V
    MAKE-SET(v)
for each edge (u, v) ∈ G.E
    if FIND-SET(u) ≠ FIND-SET(v)
        UNION(u, v)
```

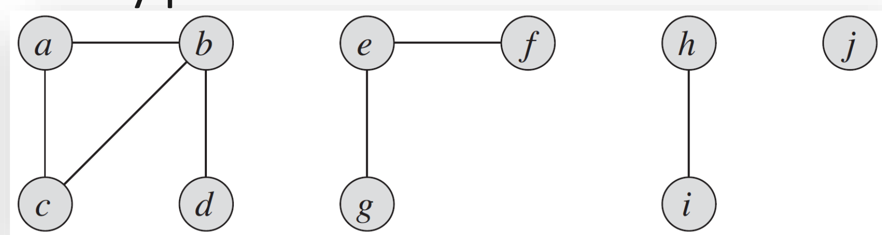
Edge processed	Collection of disjoint sets									
initial sets	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{e\}$	$\{f\}$	$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$
(b,d)	$\{a\}$	$\{b,d\}$	$\{c\}$		$\{e\}$	$\{f\}$	$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$
(e,g)	$\{a\}$	$\{b,d\}$	$\{c\}$		$\{e,g\}$	$\{f\}$		$\{h\}$	$\{i\}$	$\{j\}$
(a,c)	$\{a,c\}$	$\{b,d\}$			$\{e,g\}$	$\{f\}$		$\{h\}$	$\{i\}$	$\{j\}$
(h,i)	$\{a,c\}$	$\{b,d\}$			$\{e,g\}$	$\{f\}$		$\{h,i\}$		$\{j\}$
(a,b)	$\{a,b,c,d\}$				$\{e,g\}$	$\{f\}$		$\{h,i\}$		$\{j\}$
(e,f)	$\{a,b,c,d\}$				$\{e,f,g\}$			$\{h,i\}$		$\{j\}$
(b,c)	$\{a,b,c,d\}$				$\{e,f,g\}$			$\{h,i\}$		$\{j\}$

Example…

- Finally, the procedure SAME-COMPONENT can determine whether two vertices are in the same connected component

  A component, sometimes called a connected component, of an undirected graph is a subgraph in which any two vertices are connected to each other by paths

```
SAME-COMPONENT(u, v)
  if FIND-SET(u) == FIND-SET(v)
      return TRUE
  else return FALSE
```



Edge processed	Collection of disjoint sets									
initial sets	{a}	{b}	{c}	{d}	{e}	{f}	{g}	{h}	{i}	{j}
(b,d)	{a}	{b,d}	{c}		{e}	{f}	{g}	{h}	{i}	{j}
(e,g)	{a}	{b,d}	{c}		{e,g}	{f}		{h}	{i}	{j}
(a,c)	{a,c}	{b,d}			{e,g}	{f}		{h}	{i}	{j}
(h,i)	{a,c}	{b,d}			{e,g}	{f}		{h,i}		{j}
(a,b)	{a,b,c,d}				{e,g}	{f}		{h,i}		{j}
(e,f)	{a,b,c,d}				{e,f,g}			{h,i}		{j}
(b,c)	{a,b,c,d}				{e,f,g}			{h,i}		{j}

Questions?

kychen@mail.ntust.edu.tw